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# LINEAR ALGEBRA LEARNING IN SOLVING SYSTEM OF LINEAR EQUATIONS MATTER EXPRESSED IN MATRIX EQUATION USING CONSTRUCTIVIST APPROACH IN IMPROVING PROBLEM SOLVING 

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#### Abstract

Linear algebra has a characteristic of competency hierarchically; it means that the competency in the past is as the requirement to get the next competency. With such characteristic needed constructivist learning approach, which has characteristics such as: (1) Knowledge is actively constructed by the students and knowing is an adaptive process (Symeou, L., 2005: 18; Cristie, 2005), (2) learning is individual interpretation of the world, emphasizing problem solving and understanding, use of authentic tasks, experience, and settings as well as the contents of learning are presented holistically not separated (Cristie, 2005), (3) The pressure of learning process is on students and emphasized on the process rather than outcomes, (4) curriculum emphasizes student participation and (5) the teacher is a facilitator (Aunurrahman, 2009: 25; Soeparno, 1997: 66). Linear algebra learning in solving system of linear equations matter expressed in matrix equation using constructivist approach that emphasizes the process of student to solve problems based on the experience that has been gained, students will experience the process of identifying, formulating procedures and steps to resolve, implementing strategies to solve problems, interpreting the results in accordance of an early problems and using the mathematical concepts that have been acquired or learned as an individual experience to solve the problems encountered. Given the process, it will give students the ability in solving problems. This is in accordance with opinion of Sumarmo (in Iqbal, 2010: 13) that the ability of solving mathematical problems, characterized by the ability to identify these elements.


Keywords: constructivist, linear algebra, problem solving

## INTRODUCTION

Linear Algebra course is the main course in Mathematical Studies Program. The course has a code11028 TMT is learn about the system of linear equations (SPL) and the matrix, the determinant of the matrix, vectors in the $R^{2}$ and $R^{3}$, vector spaces, linear transformations, eigenvalues and eigen vectors.

When studying systems of linear equations, and competencies that will be gained by student is to determine the general shape of, augmented, perform Elementary row operation, finish SPL with Gaussian elimination, determine the set of completion of homogeneous SPL, and explain matrix and its operation, determine the elementary matrix, and using the inverse
of the SPL. In the SPL and this matrix, the basic competency is students should be able to finish SPL by using the matrix equation. Competence of students that obtained when studying the determinants is able to calculate the determinant by rows reduction, using the properties of determinants, determine cofactor expansion, using the Cremer rules. Expected competencies when studying vectors in the space are explaining geometric, determining a vector norm, performing point multiplication (projection), performing the cross multiplication, and determining lines and plane in space. For competencies gained while studying the linear transformation is defining the linear transformation, determining kernel and range, determining linear transformation from $R^{n}$ to $R^{m}$, determining linear transformation matrix, and determining similarity. While
studying the eigenvalues and eigen vectors the competencies acquired is determining eigenvalues and eigenvectors, performing matrix diagonalization, and performing orthogonal diagonalization.

The competencies in Linear Algebra courses have a relation hierarchically, meaning that previously acquired competence is a requirement to get the next competency. Of course, to master the material, that students should master material studied previously. This is evidenced when completing the system of linear equations using matrix inverse, then it should be able to define the inverse. To determine the inverse matrix with number of rows or columns more than $3 \times 3$ then it should be able to perform elementary row operations previously learned. And to perform elementary row operations would have to know the rules and theorems related. And so on, a lot of problem solving that can be solved with a problem solving mastery before.

The success of Liner Algebra courses into this part of mathematics that can be seen from the problem solving ability of students. This is accordance with the results of mathematical learning formulated by the National Council of Teachers of Mathematics (NCTM: 2000):
"Outline the learners should learn mathematics through understanding and actively build prior knowledge. Realizing this, the learning of mathematics formulated five general purposes: first, learning to communicate, secondly, learning to reason; third, learning to solve problems; fourth, learning to associate the idea, and the fifth, the formation of positive attitudes towards mathematics ".

The problem is something that cannot be separated from human beings, so that mathematical problem solving ability is intended ability in learning mathematics. Laster (Branca: 1980) in Sugiman et al (2009: 179) states that "Problem solving is the heart of mathematics" which means that the heart of mathematics is problem solving. Furthermore, NCTM (National Council of teachers of Mathematics) asserted that problem solving ability is one important aspect in making man into literat in mathematics (Romberg: 1994) in Sugiman et al (2009: 179). From the opinion above it can be stated that the problem solving is important in learning mathematics.

By looking at the characteristics of linear algebra which has hierarchy materials, to gain the next competencies so they must master the previously learned competencies. And to solve the problems encountered,
the students should be able to solve the problem in the previous competencies. A lecturer needs to pay attention the learning process by always linking and using what you already have to solve the problems encountered. Hierarchy material characteristics are very appropriate to use constructivist learning model. According to constructivist theory, learning is an active process and that knowledge is constructed from experience and learning is individual interpretation of the world (Christie, 2005; Symeou, L., 2005). In the constructivist model of learning, knowledge is constructed by learners through experience, social interaction and the real world.

From the background above, the author wanted to develop linear algebra learning in solving system of linear equations matter expressed in matrix equation using constructivist approach in improving problem solving.

## Constructivist Approach

Terminologically, the meaning of constructivist derived from the word "to construct" which means "form". Constructivism is a philosophical school that has the view that the knowledge that we have is the result of construction or formation ourselves. Constructivism holds that knowledge is an individual acquisition through active involvement in the learning process to take. Anita Woolfolk (2005: 323) argues constructivist approach as "... learning that emphasizes the active role of students in building understanding and the meaning and interpretation of information and events experienced".

According to Baharuddin (2012: 116), the philosophical study according to constructivist theory is to build a knowledge bit by bit, which then results are expanded through a limited context and not suddenly. Therefore, constructivism is actually a cornerstone of thinking (philosophy) of contextual learning. Knowledge is not a set of facts, concepts or rules that are ready to be retrieved or remembered. Humans must construct knowledge and give meaning through real experience. Therefore, Slavin (1994) stated that in the process of teaching and learning, the student must actively engage students into the center of activity.

There are a few things that need attention in constructivist learning, (Jauhar, 2011: 37), namely (1) prioritize the learning that is evident in the context of the relevant (contextual), (2) prioritizing process, (3) embed learning within the context of the experience social, (4) learning is done in an effort to construct experience.

The constructivist learning principles, among others: (1) Knowledge is actively constructed by the students and knowing is an adaptive process (Symeou,
L., 2005: 18; Christie, 2005), (2) learning is individual interpretation of the world, emphasizing problem solving and understanding, use of authentic tasks, experience, and settings as well as the contents are presented in a holistic learning are not separate-separate (Christie, 2005), (3) the pressure is on students' learning process and emphasis on process rather than outcomes, (4) curriculum emphasizes student participation and (5) the teacher is a facilitator (Aunurrahman, 2009: 25; Soeparno, 1997: 66).

Because the core of constructivist learning that knowledge is built by the students through experiences, social interactions and the real world, then the implementation should use active learning strategies (active learning), cooperative learning (cooperative learning) and contextual learning (contextual learning).

## Problem Solving

The problem is essentially the gap between the real situation and the desired conditions, or between reality and what is desired. The gap manifests itself in the form of complaints, anxiety, restlessness, and anxiety. Problem solving is the process of thinking and find solutions to those problems.

Ruseffendi (in Nuraeni 2011: 12) argues that an issue that is a problem for students if: (1) the student has the ability to solve them, both mental stability and knowledge, (2) the issue was not known by them so that the student has not known the procedure to finish it, (3) something that is a problem for him, if he had intention to solve it. Thus, the questions that the problem is the questions that is not routine. According Suherman (in Nuraeni 2011: 12) problems in mathematics is not a routine problem, meaning that the solution method is not known.

According to Polya (in Suherman, 2003: 91), solutions of problem solving problem containing fourstep phase of completion, which is understanding the problem, planning a problem, solve the problem according to plan, and to check back on all the steps that have been done.

Darhim (2004: 18) states that indicators of achievement can be seen in the problem solving ability of specific learning objectives (TPK) in the teaching and learning activities. The indicators of the topic are to develop a basic math test used to determine students' mathematical problem solving ability.

Indicators of problem solving according to NCTM (2000) for secondary education, namely:

1. Apply and adapt a variety of strategies to solve problems.
2. Solve problems that Arise in mathematics and Reviews those involving mathematics in other contexts.
3. Build new mathematical knowledge through problem solving.
4. Monitor and reflect on the process of mathematical problem solving.

The Sumarmo (in Iqbal, 2010: 13) argues some indicator of the type of mathematical thinking competence, in particular mathematical problem solving, including:

1. Identifying the elements that are known,
2. Formulating a mathematical problem or to develop a mathematical model,
3. Applying strategies to solve a variety of problems inside or outside of mathematics,
4. Interpreting the results as initial problems, and
5. Using mathematics meaningfully.

## RESULTS AND DISCUSSION

Implementation of Linear Algebra Learning in Solving System of Linear Equations Matter Expressed in Matrix Equation Using Constructivist Approach in Improving Problem Solving

1. Linear Algebra Learning in Solving System of Linear Equations Matter Expressed in Matrix Equation Using Constructivist Approach

In the previous discussion has made clear that the core of constructivist learning is the knowledge built by students experience, social interaction and the real world, then the implementation should use active learning strategies (active learning), cooperative learning (cooperative learning) and contextual learning (contextual learning ). Selection of appropriate learning approach to the material will provide high competence associated with material learned.

The steps of the linear algebra learning in solving system of linear equations matter expressed in matrix equation using constructivist approach are as follows:
a. Doing apperception is reviewing the question and answer material related to the solving SPL expressed in matrix equations, including asking about; 1) how to create a system of linear equations of contextual issues? 2) How to express the SPL into a matrix form enlarged? 3) How do the ways of elementary row operations?
b. Lecturer delivers the purpose of learning that is completing the system of linear equations using
matrix equations by means of discussions with constructivist approach.
c. Lecturer divides students into several small groups consist of 3-5 students.
d. Students are given the contextual issues related to the system of linear equations.

For example:
Review the following system of linear equations, and determine the value $X_{1}, X_{2}, X_{3}$ by using matrix equation!

$$
\begin{aligned}
& X_{1}+2 X_{2}+3 X_{3}=5 \\
& 2 X_{1}+5 X_{2}+3 X_{3}=3 \\
& X_{1}+8 X_{3}=17
\end{aligned}
$$

e. In the group, each student individually asked to identify the completion procedure. If the individually is difficult, it may be questioned to others in his group.

1) Writing the system of linear equations in matrix form, so it will find the components of the matrix that is written in equation system $\mathrm{AX}=$ B, where

$$
A=\left[\begin{array}{lll}
1 & 2 & 3 \\
2 & 5 & 3 \\
1 & 0 & 8
\end{array}\right] X=\left[\begin{array}{l}
X_{1} \\
X_{2} \\
X_{3}
\end{array}\right]
$$

$$
B=\left[\begin{array}{c}
5 \\
3 \\
17
\end{array}\right]
$$

2) Inversing matrix A. In this case the student is stressed to recall past experiences to find the inverse matrix A by using the way of Elementary Operations number is obtained. In this way students are required to perform back elementary row operation of matrices A and eventually in the inverse of matrix $A$ can be as follows.

$$
A^{-1}=\left[\begin{array}{ccc}
-40 & 16 & 9 \\
13 & -5 & -3 \\
5 & -2 & -1
\end{array}\right]
$$

3) Students are asked to find the value. To find this value also re-emphasized the students to find themselves with the experience that has been done in a matrix multiplication.

$$
\mathrm{AX}=\mathrm{B}
$$

$$
\begin{aligned}
& X=A^{-1} \cdot B \\
& {\left[\begin{array}{l}
X_{1} \\
X_{2} \\
X_{3}
\end{array}\right]=\left[\begin{array}{ccc}
-40 & 16 & 9 \\
13 & -5 & -3 \\
5 & -2 & -1
\end{array}\right]\left[\begin{array}{c}
5 \\
3 \\
17
\end{array}\right]}
\end{aligned}
$$

So that would be obtained following the completion of the set

$$
\mathrm{HP}=\{1,-1,2\}
$$

f. Students are asked to present the results of his work with the lecturer choose the student's names individually. Another student were asked to respond in accordance with the results of his work that ultimately happen a process of discussion to find the right answer to the problem. In this case the lecturer as facilitator facilitating the discovery of processes related to the answer to the problems that exist.
g. Lecturer together with students concludes material of solving system of linear equations using matrix equation
h. Lecturer reflects to see where the process is deemed necessary to do deepening, so that students can actually solve the problems the system of linear equations.
i. Lecturer provides contextual issues to be resolved again, as deepening and follow-up.

## 2. Linear Algebra Learning in Solving System of Linear Equations Matter Expressed in Matrix Equation Using Constructivist Approach in Improving Problem Solving

Linear algebra learning in solving system of linear equations matter expressed in matrix equation using constructivist approach that emphasizes the process of students to solve problems based on the experience that has been gained, students will experience the process as follows:
a. Identifying the known elements of the system of linear equations to be expressed into the form of a matrix equation system,
b. Students formulate procedures and steps to resolve the SPL question with the matrix equation that emphasizes the process and student experience. The steps taken is to make the SPL into the system matrix equation, determine the inverse by elementary row operations, matrix multiplication, and define the set of completion,
c. Implementing strategies to solve a variety of problems, this is done in identifying the completion of the SPL by using matrix equations.

Implementation of the strategy is also done when finding the inverse by using the OBE,
d. Interpreting the results as initial problems with determining the set of completion from linear equations system which has been given,
e. Using a mathematical concept that has been obtained or learned as an individual experience to solve the problems faced such as to finishing SPL using equations matrix system.

Given the process, will give students the ability to solve problems. This is in accordance with the opinion of Sumarmo (in Iqbal, 2010: 13) that the ability of solving mathematical problems, characterized by the ability to identify the elements that are known, to formulate a mathematical problem or to develop a mathematical model, implement strategies to solve a variety of problems inside or outside mathematics, interpret the results as initial problems, and using mathematics meaningfully.

## CONCLUSION

Linear algebra learning in solving system of linear equations matter expressed in matrix equation using constructivist approach that emphasizes the process of students to solve problems based on the experience that has been gained, students will experience the process of identifying, formulating procedures and steps to resolve, implement strategies to solve problems, interpret results as initial problems and use mathematical concepts that have been acquired or learned as an individual experience to solve the problems encountered. Given the process, will give students the ability to solve problems.

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